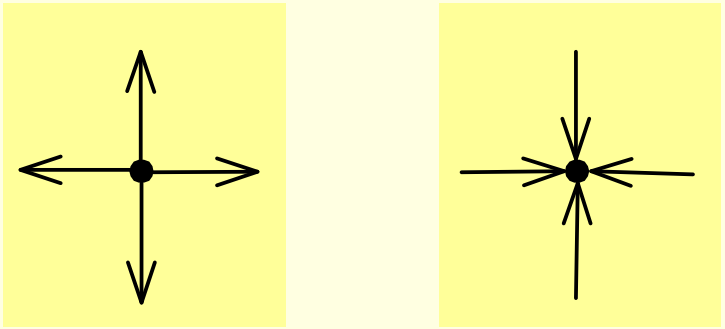
Levi-Civita Epsilon 4D (3DH)

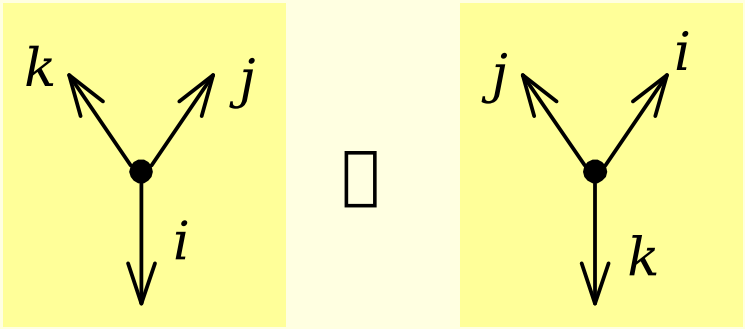
$e_{ijkl} = +1$ if $ijkl$ is an even permutation of 1234

$e_{ijkl} = -1$ if $ijkl$ is an odd permutation of 1234

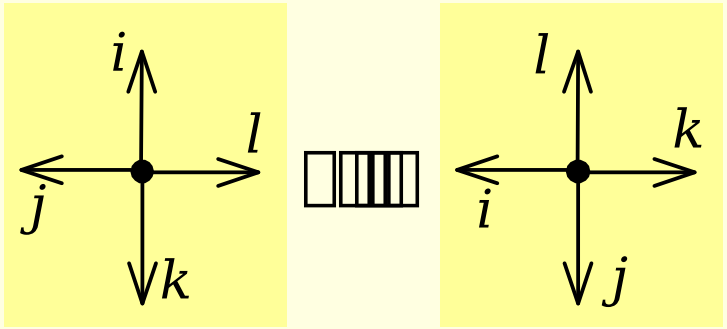
$e_{ijkl} = 0$ otherwise



Anti-Symmetry of Epsilons

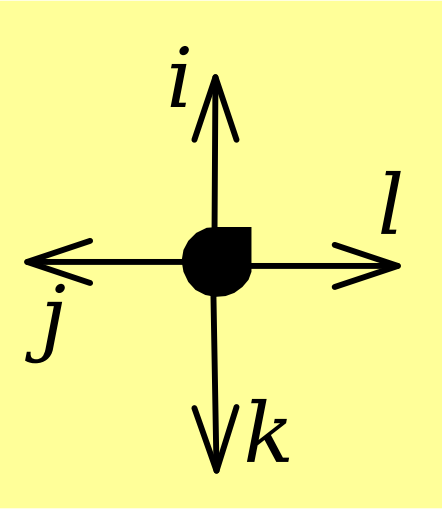


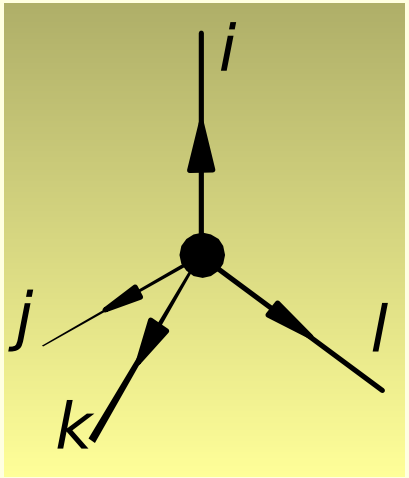
ε_{ijk} = ε_{kij}



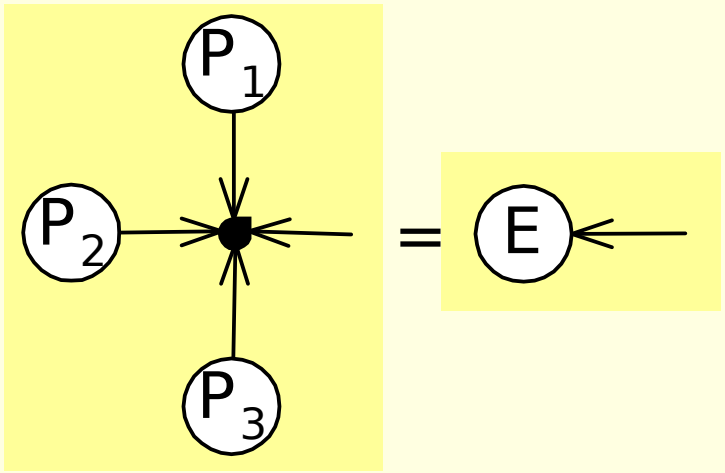
ε_{ijkl} = $-\varepsilon_{lijk}$

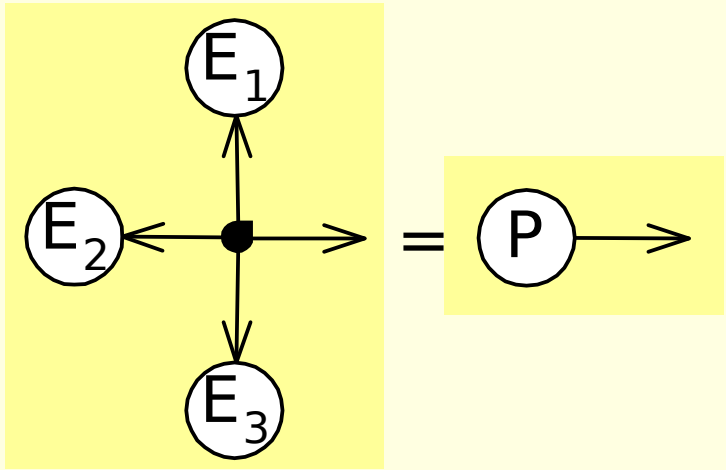
Notation for Anti-Symmetry of 4D Epsilon



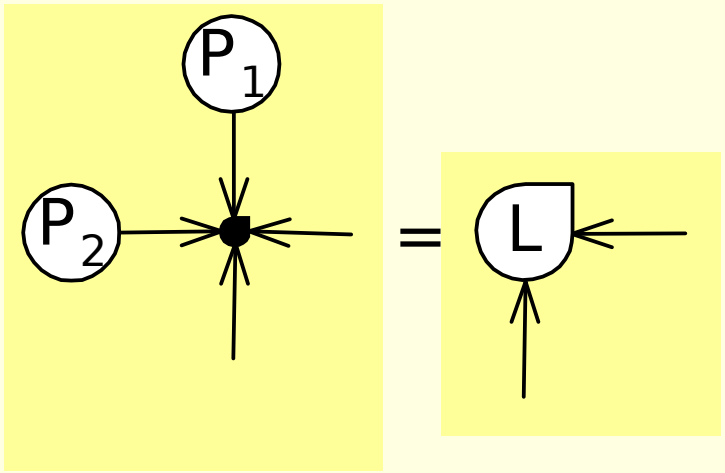


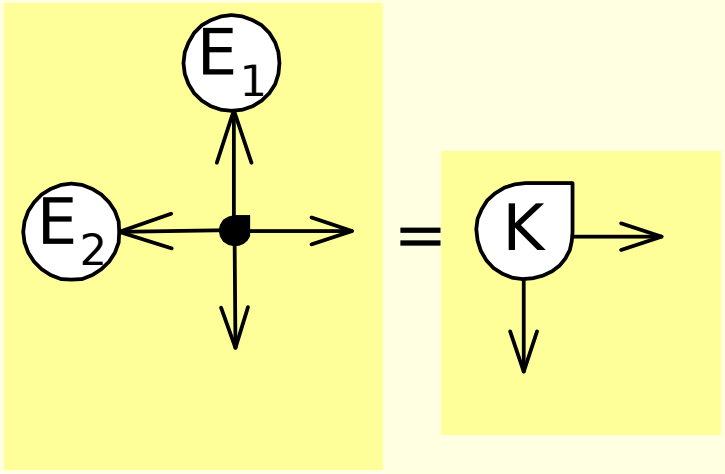
3 Points and 3 Planes



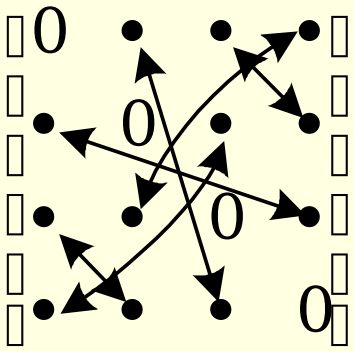


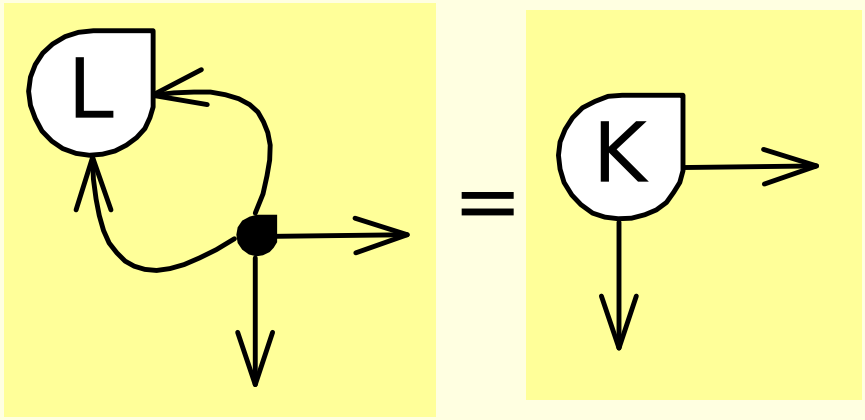
2 Points and 2 Planes = Line



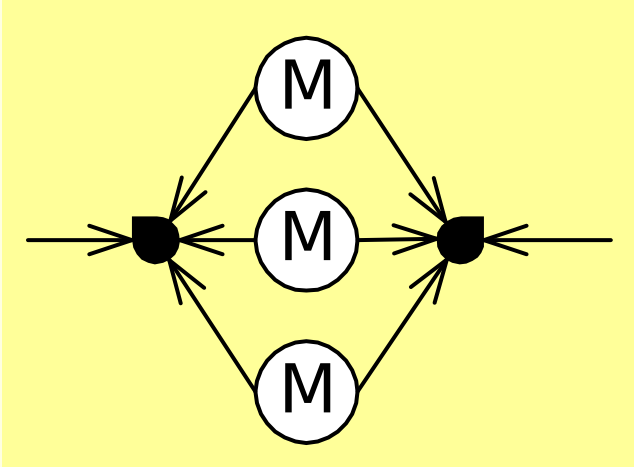


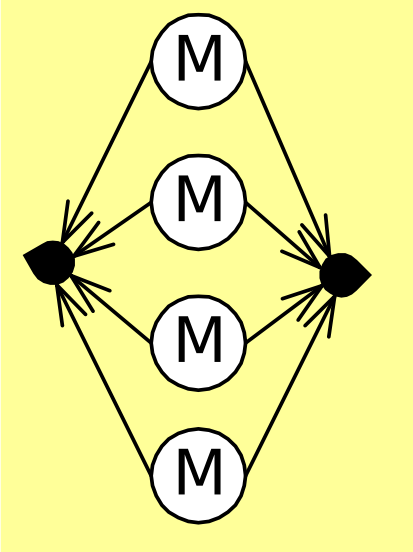
Relation between 2 Line Tensors





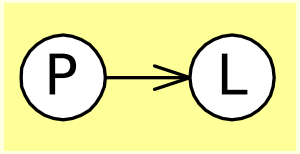
Adjoint and Determinant



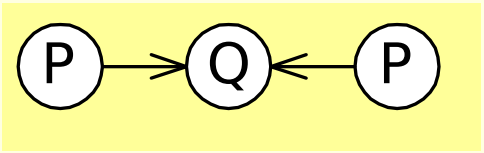


2D (1DH) Homogeneous Polynomials

$$Ax + Bw = \begin{bmatrix} x & w \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

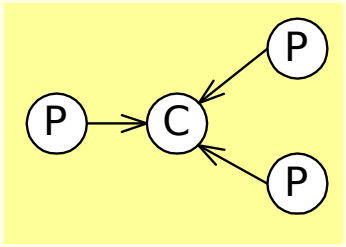


$$Ax^2 + 2Bxw + Cw^2 = \begin{bmatrix} x & w \end{bmatrix} \begin{bmatrix} A & B \\ B & C \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix}$$

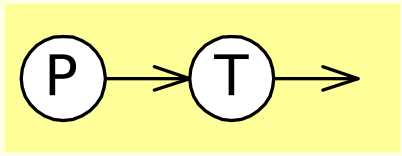


$$Ax^3 + 3Bx^2w + 3Cxw^2 + Dw^3$$

$$= \begin{bmatrix} x & w \end{bmatrix} \begin{bmatrix} A & B & B & C \\ B & C & C & D \end{bmatrix} \begin{bmatrix} x & x \\ w & w \end{bmatrix}$$



$$\begin{bmatrix} x & w \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} x' & w' \end{bmatrix}$$



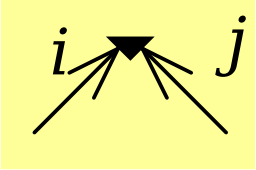
2D (1DH) Levi-Civita Epsilon

$$\varepsilon_{12} = 1$$

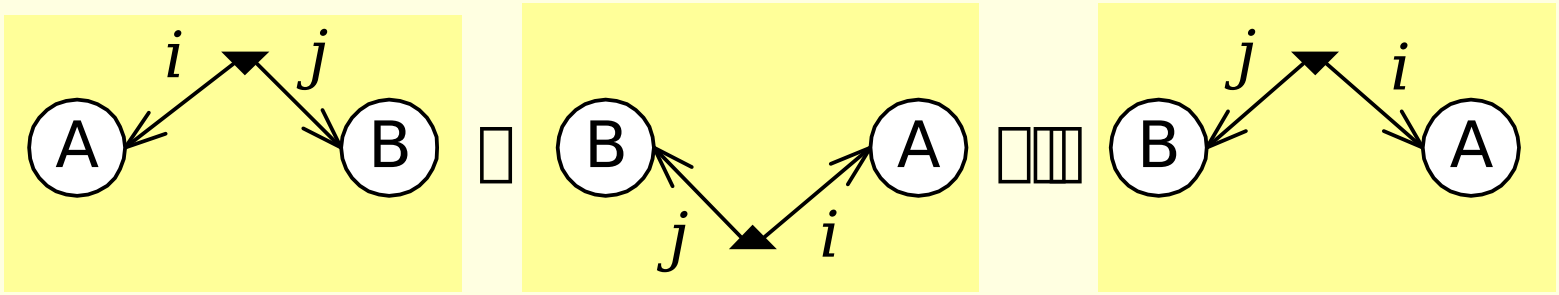
$$\varepsilon_{21} = -1$$

$$\varepsilon_{ij} = 0 \quad otherwise$$

$$\varepsilon = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$



Anti-Symmetry of 2D Epsilon

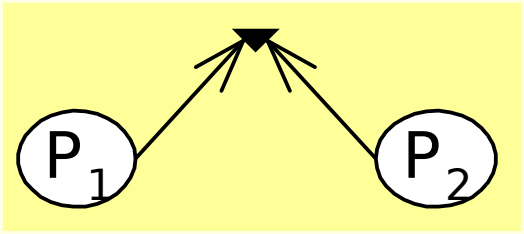


Homogeneous Equality

$$\frac{x_1}{w_1} = \frac{x_2}{w_2}$$

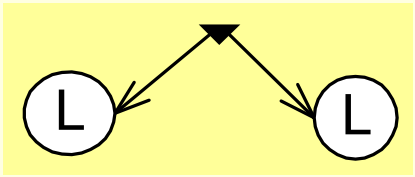
$$x_1 w_2 - w_1 x_2 = 0$$

$$\begin{bmatrix} x_1 & w_1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ w_2 \end{bmatrix} = 0$$

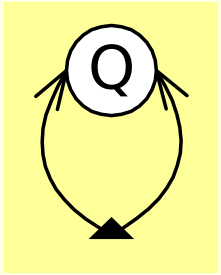


$$= 0$$

Identities

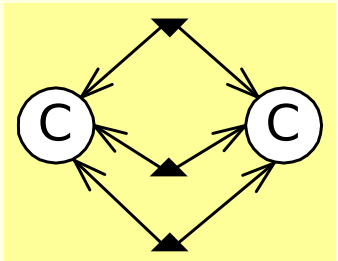


$= 0$

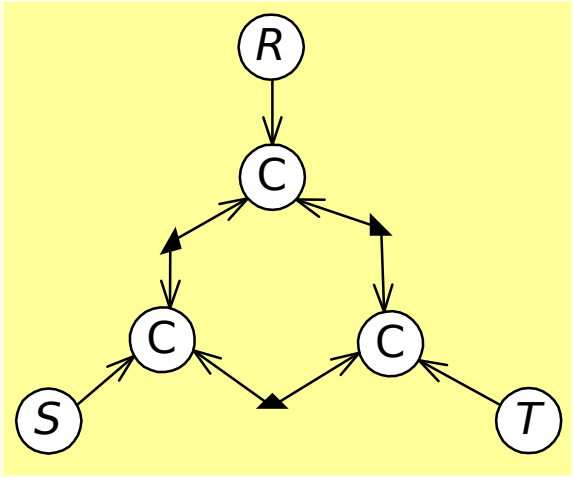


$= 0$

$$trace\begin{pmatrix} A & B & 1 \\ B & C & 0 \end{pmatrix} = trace\begin{pmatrix} B & A \\ C & B \end{pmatrix}$$

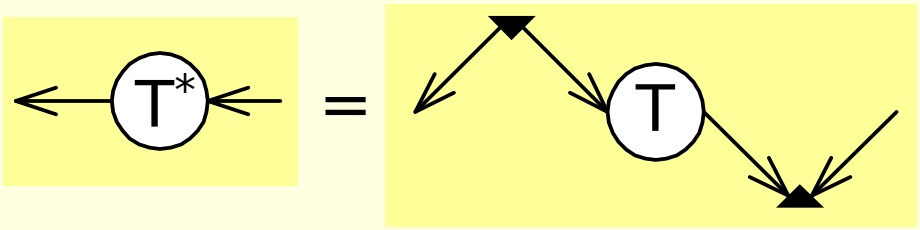


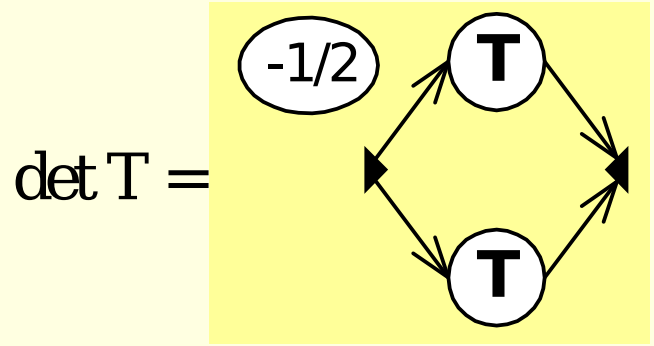
$= 0$



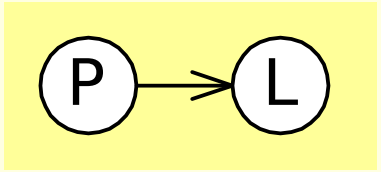
$= 0$

Adjoint and Determinant



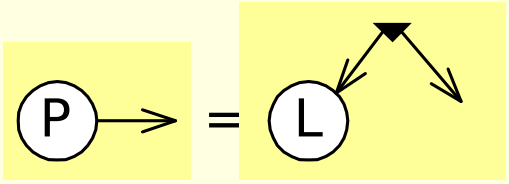


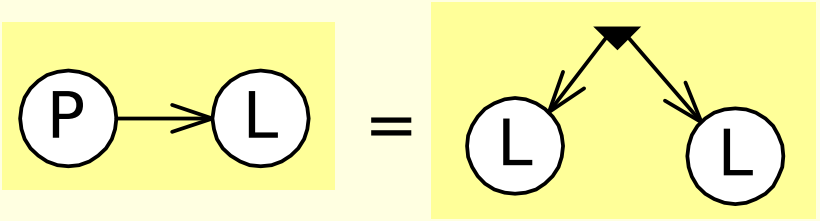
Solving Linear Equation

$$Ax + Bw = \begin{bmatrix} x & w \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} =$$ 

$$= 0$$

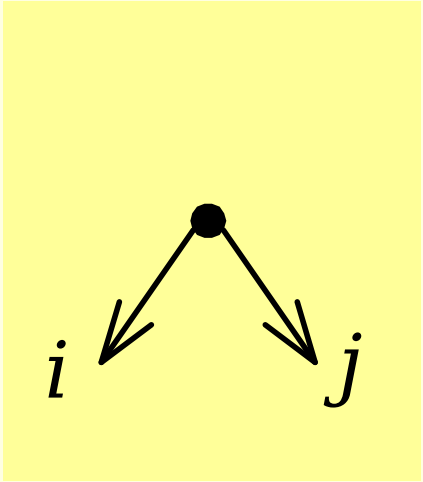
$$\begin{bmatrix} x & w \end{bmatrix} = \begin{bmatrix} -B & A \end{bmatrix}$$





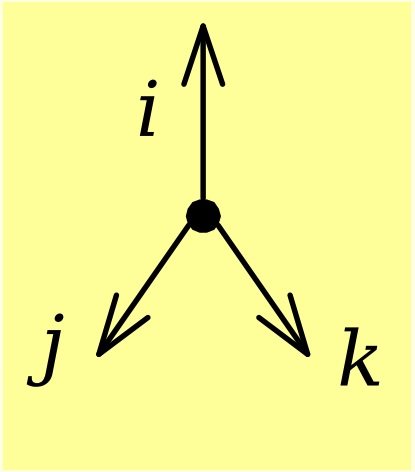
$$= 0$$

Dimensionality and Epsilon



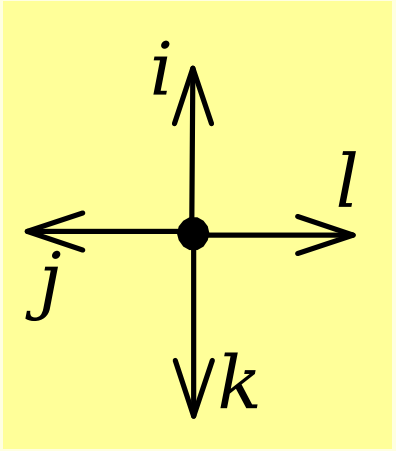
ε^{ij}

2D(1DH)



ε^{ijk}

3D(2DH)

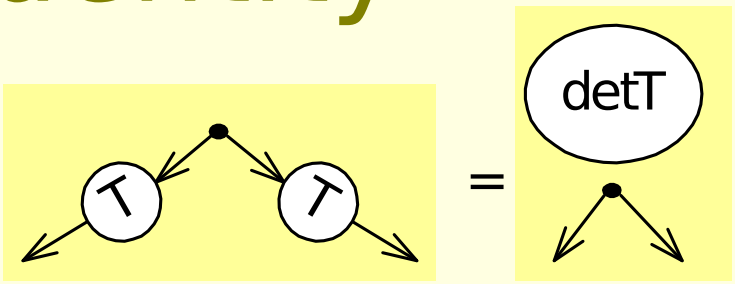


ε^{ijkl}

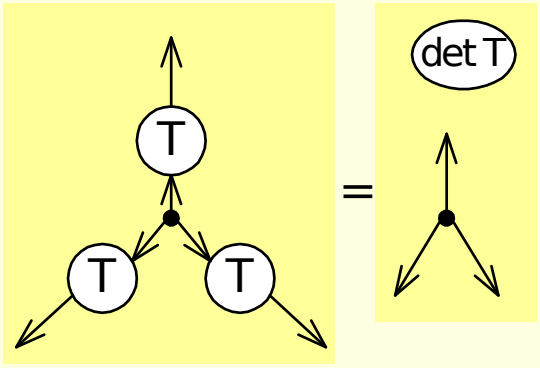
4D(3DH)

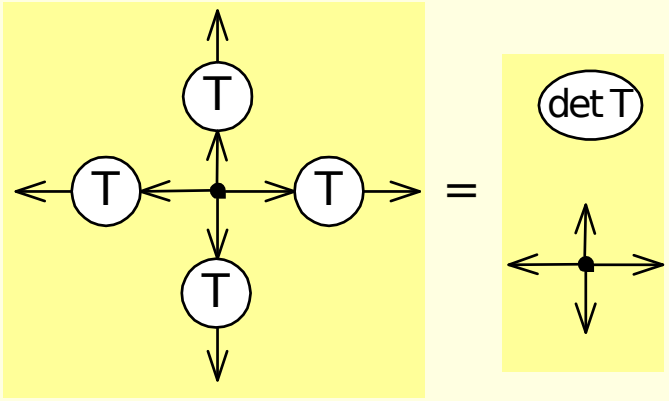
MAJOR PUNCHLINE

Another Determinant Identity

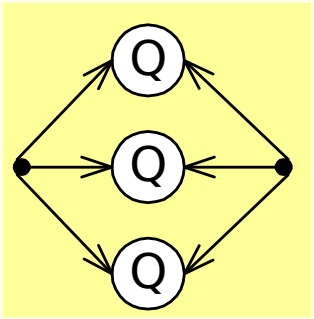


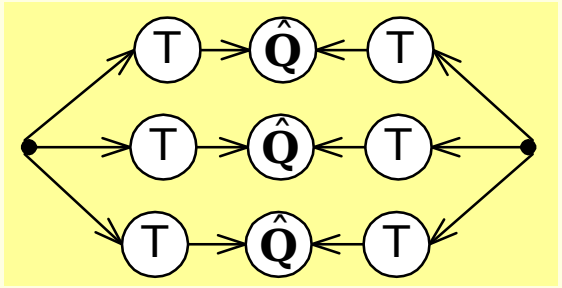
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} c & d \\ a & b \end{bmatrix} \qquad \begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & ad-bc \\ bc-ad & 0 \end{bmatrix}$$

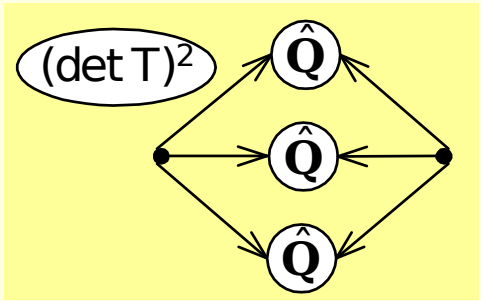




Transformationally Invariant Diagrams







Invariants of Cubic Curve